

37. A customer is offered an investment where interest is calculated according to the following force of interest:

$$\delta_t = \begin{cases} 0.02t & 0 \leq t \leq 3 \\ 0.045 & 3 < t \end{cases}$$

The customer invests 1000 at time $t = 0$.

What nominal rate of interest, compounded quarterly, is earned over the first four-year period?

- (A) 3.4%
- (B) 3.7%
- (C) 4.0%
- (D) 4.2%
- (E) 4.5%

53. At time 0, K is deposited into Fund X, which accumulates at a force of interest $\delta_t = 0.006t^2$. At time m , $2K$ is deposited into Fund Y, which accumulates at an annual effective interest rate of 10%.

At time n , where $n > m$, the accumulated value of each fund is $4K$.

Determine m .

- (A) 1.6
- (B) 2.4
- (C) 3.8
- (D) 5.0
- (E) 6.2

12. Bruce and Robbie each open up new bank accounts at time 0 . Bruce deposits 100 into his bank account, and Robbie deposits 50 into his. Each account earns an annual effective discount rate of d .

The amount of interest earned in Bruce's account during the 11th year is equal to X . The amount of interest earned in Robbie's account during the 17th year is also equal to X .

Calculate X .

- (A) 28.0
- (B) 31.3
- (C) 34.6
- (D) 36.7
- (E) 38.9

45. At time $t = 0$, 1 is deposited into each of Fund X and Fund Y . Fund X accumulates at a force of interest $\delta_t = \frac{t^2}{k}$. Fund Y accumulates at a nominal rate of discount of 8% per annum convertible semiannually.

At time $t = 5$, the accumulated value of Fund X equals the accumulated value of Fund Y .

Determine k .

- (A) 100
- (B) 102
- (C) 104
- (D) 106
- (E) 108

49. Tawny makes a deposit into a bank account which credits interest at a nominal interest rate of 10% per annum, convertible semiannually.

At the same time, Fabio deposits 1000 into a different bank account, which is credited with simple interest.

At the end of 5 years, the forces of interest on the two accounts are equal, and Fabio's account has accumulated to Z .

Determine Z .

- (A) 1792
- (B) 1953
- (C) 2092
- (D) 2153
- (E) 2392

1. Ernie makes deposits of 100 at time 0, and X at time 3 . The fund grows at a force of

interest $\delta_t = \frac{t^2}{100}, t > 0$.

The amount of interest earned from time 3 to time 6 is X .

Calculate X .

- (A) 385
- (B) 485
- (C) 585
- (D) 685
- (E) 785

24. David can receive one of the following two payment streams:

- (i) 100 at time 0, 200 at time n , and 300 at time $2n$
- (ii) 600 at time 10

At an annual effective interest rate of i , the present values of the two streams are equal.

Given $v^n = 0.75941$, determine i .

- (A) 3.5%
- (B) 4.0%
- (C) 4.5%
- (D) 5.0%
- (E) 5.5%

47. Project P requires an investment of 4000 at time 0 . The investment pays 2000 at time 1 and 4000 at time 2 .

Project Q requires an investment of X at time 2 . The investment pays 2000 at time 0 and 4000 at time 1 .

Using the net present value method and an interest rate of 10%, the net present values of the two projects are equal.

Calculate X .

- (A) 5400
- (B) 5420
- (C) 5440
- (D) 5460
- (E) 5480

1. Bruce deposits 100 into a bank account. His account is credited interest at a nominal rate of interest i convertible semiannually.

At the same time, Peter deposits 100 into a separate account. Peter's account is credited interest at a force of interest of δ .

After 7.25 years, the value of each account is 200.

Calculate $(i - \delta)$.

- (A) 0.12%
- (B) 0.23%
- (C) 0.31%
- (D) 0.39%
- (E) 0.47%

12. Eric deposits X into a savings account at time 0, which pays interest at a nominal rate of i , compounded semiannually.

Mike deposits $2X$ into a different savings account at time 0, which pays simple interest at an annual rate of i .

Eric and Mike earn the same amount of interest during the last 6 months of the 8th year.

Calculate i .

- (A) 9.06%
- (B) 9.26%
- (C) 9.46%
- (D) 9.66%
- (E) 9.86%

50. Jeff deposits 10 into a fund today and 20 fifteen years later. Interest is credited at a nominal discount rate of d compounded quarterly for the first 10 years, and at a nominal interest rate of 6% compounded semiannually thereafter. The accumulated balance in the fund at the end of 30 years is 100.

Calculate d .

- (A) 4.33%
- (B) 4.43%
- (C) 4.53%
- (D) 4.63%
- (E) 4.73%

7. Mike receives cash flows of 100 today, 200 in one year, and 100 in two years. The present value of these cash flows is 364.46 at an annual effective rate of interest i .

Calculate i .

- (A) 10%
- (B) 11%
- (C) 12%
- (D) 13%
- (E) 14%

13. At a nominal interest rate of i convertible semi-annually, an investment of 1000 immediately and 1500 at the end of the first year will accumulate to 2600 at the end of the second year.

Calculate i .

- (A) 2.75%
- (B) 2.77%
- (C) 2.79%
- (D) 2.81%
- (E) 2.83%

18. A store is running a promotion during which customers have two options for payment.

Option one is to pay 90% of the purchase price two months after the date of sale.

Option two is to deduct $X\%$ off the purchase price and pay cash on the date of sale.

A customer wishes to determine X such that he is indifferent between the two options when valuing them using an effective annual interest rate of 8%.

Which of the following equations of value would the customer need to solve?

(A) $\left(\frac{X}{100}\right)\left(1 + \frac{0.08}{6}\right) = 0.90$

(B) $\left(1 - \frac{X}{100}\right)\left(1 + \frac{0.08}{6}\right) = 0.90$

(C) $\left(\frac{X}{100}\right)(1.08)^{1/6} = 0.90$

(D) $\left(\frac{X}{100}\right)\left(\frac{1.08}{1.06}\right) = 0.90$

(E) $\left(1 - \frac{X}{100}\right)(1.08)^{1/6} = 0.90$

19. Calculate the nominal rate of discount convertible monthly that is equivalent to a nominal rate of interest of 18.9% per year convertible monthly.

- (A) 18.0%
- (B) 18.3%
- (C) 18.6%
- (D) 18.9%
- (E) 19.2%

7. A bank offers the following choices for certificates of deposit:

Term (in years)	Nominal annual interest rate convertible quarterly
1	4.00%
3	5.00%
5	5.65%

The certificates mature at the end of the term. The bank does NOT permit early withdrawals. During the next 6 years the bank will continue to offer certificates of deposit with the same terms and interest rates.

An investor initially deposits 10,000 in the bank and withdraws both principal and interest at the end of 6 years.

Calculate the maximum annual effective rate of interest the investor can earn over the 6-year period.

- (A) 5.09%
- (B) 5.22%
- (C) 5.35%
- (D) 5.48%
- (E) 5.61%

25. The parents of three children, ages 1, 3, and 6, wish to set up a trust fund that will pay X to each child upon attainment of age 18, and Y to each child upon attainment of age 21. They will establish the trust fund with a single investment of Z .

Which of the following is the correct equation of value for Z ?

- (A) $\frac{X}{v^{17} + v^{15} + v^{12}} + \frac{Y}{v^{20} + v^{18} + v^{15}}$
- (B) $3[Xv^{18} + Yv^{21}]$
- (C) $3Xv^3 + Y[v^{20} + v^{18} + v^{15}]$
- (D) $(X + Y) \frac{v^{20} + v^{18} + v^{15}}{v^3}$
- (E) $X[v^{17} + v^{15} + v^{12}] + Y[v^{20} + v^{18} + v^{15}]$

32.

An investor pays 100,000 today for a 4-year investment that returns cash flows of 60,000 at the end of each of years 3 and 4. The cash flows can be reinvested at 4.0% per annum effective.

Using an annual effective interest rate of 5.0%, calculate the net present value of this investment today.

- (A) -1398
- (B) -699
- (C) 699
- (D) 1398
- (E) 2,629

33.

You are given the following information with respect to a bond:

- (i) par value: 1000
- (ii) term to maturity: 3 years
- (iii) annual coupon rate: 6% payable annually

You are also given that the one, two, and three year annual spot interest rates are 7%, 8%, and 9% respectively.

Calculate the value of the bond.

- (A) 906
- (B) 926
- (C) 930
- (D) 950
- (E) 1000

60.

You are given the following information about a loan of L that is to be repaid with a series of 16 annual payments:

- (i) The first payment of 2000 is due one year from now.
- (ii) The next seven payments are each 3% larger than the preceding payment.
- (iii) From the 9th to the 16th payment, each payment will be 3% less than the preceding payment.
- (iv) The loan has an annual effective interest rate of 7%.

Calculate L .

- (A) 20,689
- (B) 20,716
- (C) 20,776
- (D) 21,147
- (E) 22,137

61.

The annual force of interest credited to a savings account is defined by

$$\delta_t = \frac{\frac{t^2}{100}}{3 + \frac{t^3}{150}}$$

with t in years. Austin deposits 500 into this account at time 0.

Calculate the time in years it will take for the fund to be worth 2000.

- (A) 6.7
- (B) 8.8
- (C) 14.2
- (D) 16.5
- (E) 18.9

76.

Consider two 30-year bonds with the same purchase price. Each has an annual coupon rate of 5% paid semiannually and a par value of 1000.

The first bond has an annual nominal yield rate of 5% compounded semiannually, and a redemption value of 1200.

The second bond has an annual nominal yield rate of j compounded semiannually, and a redemption value of 800.

Calculate j .

- (A) 2.20%
- (B) 2.34%
- (C) 3.53%
- (D) 4.40%
- (E) 4.69%

77.

Lucas opens a bank account with 1000 and lets it accumulate at an annual nominal interest rate of 6% convertible semiannually. Danielle also opens a bank account with 1000 at the same time as Lucas, but it grows at an annual nominal interest rate of 3% convertible monthly.

For each account, interest is credited only at the end of each interest conversion period.

Calculate the number of months required for the amount in Lucas's account to be at least double the amount in Danielle's account.

- (A) 276
- (B) 282
- (C) 285
- (D) 286
- (E) 288

78.

On January 1, an investment fund was opened with an initial balance of 5000. Just after the balance grew to 5200 on July 1, an additional 2600 was deposited.

The annual effective yield rate for this fund was 9.00% over the calendar year.

Calculate the time-weighted rate of return for the year.

- (A) 7.43%
- (B) 8.86%
- (C) 9.00%
- (D) 9.17%
- (E) 10.45%

79.

Bill and Joe each put 10 into separate accounts at time $t = 0$, where t is measured in years.

Bill's account earns interest at a constant annual effective interest rate of $K/25$, $K > 0$.

Joe's account earns interest at a force of interest, $\delta_t = \frac{1}{K + 0.25t}$.

At the end of four years, the amount in each account is X .

Calculate X .

- (A) 20.7
- (B) 21.7
- (C) 22.7
- (D) 23.7
- (E) 24.7

94.

A couple decides to save money for their child's first year college tuition.

The parents will deposit 1700 n months from today and another 3400 $2n$ months from today.

All deposits earn interest at a nominal annual rate of 7.2%, compounded monthly.

Calculate the maximum integral value of n such that the parents will have accumulated at least 6500 five years from today.

- (A) 11
- (B) 12
- (C) 18
- (D) 24
- (E) 25

95.

Let S be the accumulated value of 1000 invested for two years at a nominal annual rate of discount d convertible semiannually, which is equivalent to an annual effective interest rate of i .

Let T be the accumulated value of 1000 invested for one year at a nominal annual rate of discount d convertible quarterly.

$$S/T = (39/38)^4.$$

Calculate i .

- (A) 10.0%
- (B) 10.3%
- (C) 10.8%
- (D) 10.9%
- (E) 11.1%

404.

A perpetuity provides for continuous payments. The annual rate of payment at time t is

$$\begin{cases} 1, & \text{for } 0 \leq t < 10, \\ (1.03)^{t-10} & \text{for } t \geq 10. \end{cases}$$

Using an annual effective interest rate of 6%, the present value at time $t = 0$ of this perpetuity is x .

Calculate x .

- (A) 27.03
- (B) 30.29
- (C) 34.83
- (D) 38.64
- (E) 42.41

105.

A bank agrees to lend 10,000 now and X three years from now in exchange for a single repayment of 75,000 at the end of 10 years. The bank charges interest at an annual effective rate of 6% for the first 5 years and at a force of interest $\delta_t = \frac{1}{t+1}$ for $t \geq 5$.

Calculate X .

- (A) 23,500
- (B) 24,000
- (C) 24,500
- (D) 25,000
- (E) 25,500